Emergence of deterministic Green's functions from noise generated by finite random sources

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Two-point correlation functions of sufficiently diffuse wave fields generated by uncorrelated random sources are known to approximate deterministic Green's functions between the two points. This property is utilized increasingly for passive imaging and remote sensing of the environment. Here we show that the relation between the Green's functions and the noise cross-correlation function holds under much less restrictive conditions than previously thought. It can even hold when ambient noise sources have correlation ranges large compared to the wavelength. Admissible correlation ranges are limited from above by the size of the Fresnel zone at wave propagation between the points where noise cross correlation is evaluated.

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I. INTRODUCTION

A deterministic Green's function (GF), which describes a wave field due to a point source, can be retrieved from the cross-correlation function (CCF) of ambient noise in an inhomogeneous medium [1,2]. A simple relation between GF and CCF was first derived for perfectly diffuse noise [1,3] and later extended to high-frequency noise fields generated by arbitrary smooth distributions of delta-correlated random wave sources in homogeneous [4] and inhomogeneous, moving or motionless [5] media (see reviews [6,7]). The emergence of GFs from noise CCFs forms the basis of the rapidly developing field of passive interferometric imaging, where spatial and temporal variations in the medium parameters are inferred from ambient noise recordings in a number of points [7–12]. While theoretical derivations rely on the assumption of the noise sources being delta correlated, remote sensing applications in geophysics and underwater acoustics utilize ambient noise generated by sources, such as nonlinearly interacting and breaking gravity waves on the ocean surface, the correlation length of which is not necessarily small compared to acoustic and seismic wavelengths or other relevant spatial scales in the problem. An understanding and theoretical justification of passive imaging with real-world noise due to correlated sources is considered a major outstanding problem [6].

In this paper, we abandon the assumption of noise sources being delta correlated and quantify the effect of the sources' correlations on long-range correlations of high-frequency acoustics fields in moving or motionless fluids. We demonstrate that, under rather nonrestrictive conditions imposed on the random sources, two-point correlation function of noise approximates the sum of the GFs, which describe sound propagation in opposite directions between the two points. The CCF of ambient noise is found to contain the information necessary to retrieve the flow velocity and the sound speed from acoustic measurements. The results obtained for sound waves in fluids are extended readily to other wave types.

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II. RANDOM SOURCES DISTRIBUTED IN A VOLUME

Consider acoustic waves in an inhomogeneous moving fluid with time-independent sound speed $c(\mathbf{x})$, mass density $\rho(\mathbf{x})$, and flow velocity $\mathbf{u}(\mathbf{x})$. As in [5,13], define the GF in the time domain, $G(\mathbf{x},\mathbf{y},t)$, as the acoustic pressure p at $\mathbf{x}$ due to a point source of volume velocity with density $B(\mathbf{x},t)=\delta(\mathbf{x}-\mathbf{y})\delta'(t)$, where δ is the delta function and t is time. The frequency spectrum $G(\mathbf{x},\mathbf{y},\omega)$ of the time-domain GF $G(\mathbf{x},\mathbf{y},t)$ has the meaning of the continuous wave (CW) Green's function. For the GFs G and $\tilde{G}$ in the original medium and a medium with reversed flow, i.e., with parameters $c(\mathbf{x})$, $\rho(\mathbf{x})$, and $-\mathbf{u}(\mathbf{x})$, a simple symmetry relation holds: $G(\mathbf{x},\mathbf{y},\omega)=\tilde{G}(\mathbf{y},\mathbf{x},\omega)$ [14]. Note that generally $G(\mathbf{x},\mathbf{y},\omega)\neq\tilde{G}(\mathbf{x},\mathbf{y},\omega)$ as sound propagates with different speeds in up- and down-flow directions. An arbitrary distribution $B(\mathbf{x})$ of CW sources of volume velocity generates acoustic pressure [13]

$$p(\mathbf{x},\omega)=i\omega^{-1}\int B(\mathbf{y})\tilde{G}(\mathbf{y},\mathbf{x},\omega)d^3\mathbf{y}, \qquad (1)$$

where the integral is taken over the entire domain occupied by the sources. Time dependence $\exp(-i\omega t)$ of CW waves is assumed and suppressed. Note that the GF in the integrand in Eq. (1) refers to the field in a fluid with reversed flow and has a variable receiver, rather than source, position.

Let an acoustic field be generated by random sources the density of which has zero mean and is locally homogeneous in the statistical sense,

$$\langle B(\mathbf{x})\rangle=0,\quad \langle B(\mathbf{x})B^*(\mathbf{y})\rangle=Q((\mathbf{x}+\mathbf{y})/2,\mathbf{x}-\mathbf{y}). \qquad (2)$$

Here and below, the asterisk and angular brackets designate complex conjugation and the average over the statistical ensemble of random sources, respectively. In the particular case of delta-correlated sources, $Q(\mathbf{x},\mathbf{y})=D(\mathbf{x})\delta(\mathbf{y})$. Designate l_1 and l_2 the representative spatial scales of the dependence of the correlation function Q on its first and second arguments, respectively. Then $l_2\ll l_1$ as long as the density of random sources is locally statistically homogeneous [15]. We assume that the scale l_1 is large compared to the acoustic wavelength. From Eqs. (1) and (2) it follows that the acoustic pressure field generated by the random sources has zero mean: $\langle p(\mathbf{x},\omega)\rangle=0$ and its CCF is given by

$$\langle p(\mathbf{x}_1,\omega)p^*(\mathbf{x}_2,\omega)\rangle = \omega^{-2}\int\int Q(\mathbf{y}_1,\mathbf{y}_2)\tilde{G}\left(\mathbf{y}_1+\frac{\mathbf{y}_2}{2},\mathbf{x}_1\right)\times\tilde{G}^*\left(\mathbf{y}_1-\frac{\mathbf{y}_2}{2},\mathbf{x}_2\right)d^3\mathbf{y}_1 d^3\mathbf{y}_2. \qquad (3)$$

The general expression (3) for the CCF can be simplified at high frequencies, when the ray approximation becomes applicable. Then

$$G(\mathbf{y},\mathbf{x},\omega)=\sum_n A_n(\mathbf{y},\mathbf{x})\exp[i\omega\varphi_n(\mathbf{y},\mathbf{x})], \qquad (4)$$

where A_n and φ_n are slowly varying complex amplitudes and rapidly varying eikonals of individual ray components of the field [16]. We assume that sound absorption is small at distances of the order of the wavelength. Then the eikonals φ_n can be considered real-valued, while the function A_n describes wave amplitude variation along the nth ray due to both absorption and geometrical spreading. The number of summands in the right-hand side of Eq. (4) depends on the environment and possibly on the source and receiver positions; e. g., the sum has a single term in the case of a homogeneous fluid with $|\mathbf{u}|<c$, two terms in the presence of a reflective boundary, and multiple terms in the case of a refractive waveguide in a vertically stratified fluid, with the number of terms increasing with the source-receiver horizontal separation [16].

In the small vicinity of an observation point $\mathbf{y}_1$, the GF [Eq. (4)] can be calculated by preserving only linear terms in the expansion of the eikonal in powers of the observation point displacement and neglecting variations in the ray amplitudes,

$$G(\mathbf{y}_1\pm\mathbf{y}_2/2,\mathbf{x})\approx\sum_n A_n(\mathbf{y}_1,\mathbf{x})\times\exp[i\omega\varphi_n(\mathbf{y}_1,\mathbf{x})\pm i\mathbf{k}_n(\mathbf{y}_1,\mathbf{x})\cdot\mathbf{y}_2/2]. \qquad (5)$$

Here $\mathbf{k}_n(\mathbf{y},\mathbf{x})\equiv\omega\,\partial\varphi_n(\mathbf{y},\mathbf{x})/\partial\mathbf{y}$ is the wave vector on the nth ray at the point $\mathbf{y}$. The approximation (5) is valid within the first Fresnel zone, i.e., as long as (i) the displacement $\mathbf{y}_2/2$ is small compared to the source-receiver separation $|\mathbf{x}-\mathbf{y}_1|$ and the spatial scale L of c,ρ, and $\mathbf{u}$ variation, and (ii) the second-order terms in the eikonal expansion are negligible. The latter condition is met when $ky_2^2\ll R$, where $R\sim\min(L,|\mathbf{x}-\mathbf{y}_1|)$ is the representative radius of curvature of the wave front.

Applying the GF approximation (5) to Eq. (3), we find

$$\langle p(\mathbf{x}_1,\omega)p^*(\mathbf{x}_2,\omega)\rangle \approx \omega^{-2}\sum_{n,m}\int Q_1\{\mathbf{y},[\tilde{\mathbf{k}}_n(\mathbf{y},\mathbf{x}_1)-\tilde{\mathbf{k}}_m(\mathbf{y},\mathbf{x}_2)/2]\}\times\tilde{A}_n(\mathbf{y},\mathbf{x}_1)\tilde{A}_m^*(\mathbf{y},\mathbf{x}_2)\exp[i\omega\tilde{\varphi}_n(\mathbf{y},\mathbf{x}_1)-i\omega\tilde{\varphi}_m(\mathbf{y},\mathbf{x}_2)]d^3\mathbf{y}, \qquad (6)$$

where Q_1 is the spatial spectrum of the correlation function of the random sound sources,

$$Q_1(\mathbf{y},\mathbf{q})=\int Q(\mathbf{y},\mathbf{y}_1)\exp(i\mathbf{q}\cdot\mathbf{y}_1)d^3\mathbf{y}_1, \qquad (7)$$

and, as before, tildes distinguish the quantities which refer to sound propagation in a medium with reversed flow. When separation of the points $\mathbf{x}_1$ and $\mathbf{x}_2$ is large compared to the acoustic wavelength, the leading order of the high-frequency asymptotic expansion of the integral in Eq. (6) is given by contributions of vicinities of stationary points of the rapidly oscillating exponential $\exp[i\omega\tilde{\varphi}_n(\mathbf{y},\mathbf{x}_1)-i\omega\tilde{\varphi}_m(\mathbf{y},\mathbf{x}_2)]$. At the stationary points $\tilde{\mathbf{k}}_n(\mathbf{y},\mathbf{x}_1)=\tilde{\mathbf{k}}_m(\mathbf{y},\mathbf{x}_2)$, and therefore the second argument of the function Q_1 under the integral can be replaced by zero when calculating the leading order of the asymptotic expansion. It follows then from Eqs. (4) and (6) that

$$\langle p(\mathbf{x}_1,\omega)p^*(\mathbf{x}_2,\omega)\rangle\approx\omega^{-2}\int Q_1(\mathbf{y},\mathbf{0})\tilde{G}(\mathbf{y},\mathbf{x}_1)\tilde{G}^*(\mathbf{y},\mathbf{x}_2)d^3\mathbf{y}. \qquad (8)$$

An inspection shows that the CCF [Eq. (8)] of the high-frequency noise generated by extended sources differs from the CCF of noise due to delta-correlated sources with density $D(\mathbf{y})$ only by substitution of an "effective density" $Q_1(\mathbf{y},\mathbf{0})$ for $D(\mathbf{y})$. According to Eq. (7), the effective density is the volume integral of the two-point correlation function of the density of the extended sources $Q(\mathbf{y},\mathbf{y}_1)$ over its fast argument $\mathbf{y}_1$.

In addition to the long-range cross-correlations, the correspondence between the CCFs of high-frequency noise fields generated by extended and by delta-correlated sources remains valid also in another case. The variance of the CW pressure field, which serves also as the power spectrum of the noise autocorrelation function in the time domain, is obtained by letting $\mathbf{x}_1=\mathbf{x}_2$ in Eqs. (3) and (6). The exponential $\exp[i\omega\tilde{\varphi}_n(\mathbf{y},\mathbf{x}_1)-i\omega\tilde{\varphi}_m(\mathbf{y},\mathbf{x}_1)]$ is a rapidly oscillating function of $\mathbf{y}$ when $n\neq m$, and equals unity when $n=m$. Hence, the leading-order term of the high-frequency asymptotic expansion of $\langle|p(\mathbf{x}_1,\omega)|^2\rangle$ is due to the diagonal terms in the double sum in Eq. (6). At $\mathbf{x}_1=\mathbf{x}_2$, by replacing Q_1 in the integrand in the right-hand side of Eq. (6) by $Q_1(\mathbf{y},\mathbf{0})$, one again obtains Eq. (8).

III. RANDOM SOURCES DISTRIBUTED ON A SURFACE

Consider acoustic waves in a volume Ω with a boundary $\partial\Omega$ described by the equation $\mathbf{x}=\mathbf{X}(\alpha,\beta)$. Real parameters α and β can be viewed as curvilinear coordinates on $\partial\Omega$. The noise field in Ω is generated by random CW sources of volume velocity distributed on $\partial\Omega$ with surface density e such that

$$\langle e(\alpha,\beta)\rangle = 0, \quad \langle e(\alpha_1,\beta_1)e^*(\alpha_2,\beta_2)\rangle = E\left[\mathbf{X}\left(\frac{\alpha_1+\alpha_2}{2},\frac{\beta_1+\beta_2}{2}\right);\alpha_1-\alpha_2,\beta_1-\beta_2\right]. \tag{9}$$

We assume slow dependence of the source correlation function E on the centroid coordinates $(\alpha_1+\alpha_2)/2$ and $(\beta_1+\beta_2)/2$ and its rapid dependence on the difference coordinates $\alpha_1-\alpha_2$ and $\beta_1-\beta_2$. As in the case of random volumetric sources, the acoustic pressure field generated by the surface sources [Eq. (9)] has zero mean: $\langle p(\mathbf{x},\omega)\rangle=0$, and its CCF is given by

$$\langle p(\mathbf{x}_1,\omega)p^*(\mathbf{x}_2,\omega)\rangle = \omega^{-2}\int_{\partial\Omega}\int_{\partial\Omega} \times E\{\mathbf{X}[(\alpha_1+\alpha_2)/2,(\beta_1+\beta_2)/2];\alpha_1-\alpha_2,\beta_1-\beta_2\} \times\tilde{G}[\mathbf{X}(\alpha_1,\beta_1),\mathbf{x}_1]\tilde{G}^*[\mathbf{X}(\alpha_2,\beta_2),\mathbf{x}_2]d\alpha_1 d\beta_1 d\alpha_2 d\beta_2. \tag{10}$$

For high-frequency noise, using Eq. (5) to approximate the GFs in Eq. (10), we find

$$\langle p(\mathbf{x}_1,\omega)p^*(\mathbf{x}_2,\omega)\rangle \approx \omega^{-2}\sum_{n,m}\int_{\partial\Omega}\tilde{A}_n(\mathbf{y},\mathbf{x}_1)\tilde{A}_m^*(\mathbf{y},\mathbf{x}_2) \times\exp[i\omega\tilde{\varphi}_n(\mathbf{y},\mathbf{x}_1)-i\omega\tilde{\varphi}_m(\mathbf{y},\mathbf{x}_2)] \times E_1\left(\mathbf{y};\mathbf{q}\cdot\frac{\partial\mathbf{X}}{\partial\alpha},\mathbf{q}\cdot\frac{\partial\mathbf{X}}{\partial\beta}\right)d\alpha d\beta, \quad \mathbf{y}=\mathbf{X}(\alpha,\beta),\mathbf{q} = \frac{1}{2}[\tilde{\mathbf{k}}_n(\mathbf{y},\mathbf{x}_1)-\tilde{\mathbf{k}}_m(\mathbf{y},\mathbf{x}_2)], \tag{11}$$

where E_1 is the power spectrum of the source CCF (with respect to the difference coordinates): $E_1(\mathbf{y};q_1,q_2) = \iint E(\mathbf{y};\alpha,\beta)\exp(iq_1\alpha+iq_2\beta)d\alpha d\beta$.

When separation of the points $\mathbf{x}_1$ and $\mathbf{x}_2$ is large compared to the acoustic wavelength, the leading order of the high-frequency asymptotic expansion of the integral in the right-hand side of Eq. (11) is due to contributions of the vicinities of the stationary points $\mathbf{y}_s\in\partial\Omega$ of the rapidly oscillating exponential in the integrand. At the stationary points, the vector $\mathbf{q}$ as defined in Eq. (11) is orthogonal to the surface $\partial\Omega$ [5], and hence $\mathbf{q}\cdot\partial\mathbf{X}/\partial\alpha=\mathbf{q}\cdot\partial\mathbf{X}/\partial\beta=0$. The leading term of the high-frequency asymptotic expansion of the integral is unaffected by replacing the last two arguments of the slowly varying function E_1 by their values at the stationary point [16]. This allows calculation of the double sum in Eq. (11) and leads to the following expression for the long-range CCF of noise due to extended surface sources,

$$\langle p(\mathbf{x}_1,\omega)p^*(\mathbf{x}_2,\omega)\rangle \approx \omega^{-2}\int_{\partial\Omega}E_1(\mathbf{y};0,0) \times\tilde{G}(\mathbf{y},\mathbf{x}_1)\tilde{G}^*(\mathbf{y},\mathbf{x}_2)d\alpha d\beta. \tag{12}$$

The right-hand side of Eq. (12) coincides with the exact CCF [5] of the random acoustic field due delta-correlated noise sources distributed on $\partial\Omega$ with the density $d(\mathbf{y})=E_1(\mathbf{y};0,0)$.

In the opposite case $\mathbf{x}_1=\mathbf{x}_2$, the leading-order contribution to the high-frequency asymptotic expansion of the right-hand side of Eq. (11) is due to the diagonal terms of the double sum since the exponential $\exp[i\omega\tilde{\varphi}_n(\mathbf{y},\mathbf{x}_1)-i\omega\tilde{\varphi}_m(\mathbf{y},\mathbf{x}_1)]$ is not oscillatory at $n=m$. With the vector $\mathbf{q}$ as defined in Eq. (11) being zero for the diagonal terms at $\mathbf{x}_1=\mathbf{x}_2$, one can replace E_1 by $E_1(\mathbf{y};0,0)$ in the integrand in the right-hand side of Eq. (11) without affecting the leading order of the high-frequency asymptotic expansion. Then Eq. (11) again reduces to Eq. (12), now for $\mathbf{x}_1=\mathbf{x}_2$.

IV. OTHER SOURCES AND WAVE TYPES

It is demonstrated above that autocorrelation and long-range cross correlation of an acoustic field due to spatially extended, random CW sources of volume velocity are asymptotically equivalent to, respectively, autocorrelation and long-range cross correlation of the acoustic field due to delta-correlated sources, the density of which is equal to the density of the extended sources integrated over the difference coordinates. This result can be easily generalized in several ways.

Because the mapping of the density of extended CW sources into the density of the equivalent CW delta-correlated sources is frequency-independent, the asymptotic equivalence holds for broadband random sources.

The CCF of the sum of noise fields generated by two statistically independent sets of sources equals the sum of the CCFs of the two fields. Hence, the equivalence to delta-correlated sources holds when there are both surface and volumetric extended sources.

Consider the acoustic field p generated by random CW volumetric sources of volume velocity and external force with the densities $B(\mathbf{x})$ and $\mathbf{F}(\mathbf{x})$, which have zero mean, are locally homogeneous in the statistical sense, and have finite correlation lengths:

$$\langle B(\mathbf{x})B^*(\mathbf{y})\rangle = Q\left(\frac{\mathbf{x}+\mathbf{y}}{2},\mathbf{x}-\mathbf{y}\right),\langle B(\mathbf{x})F_j^*(\mathbf{y})\rangle = R_j\left(\frac{\mathbf{x}+\mathbf{y}}{2},\mathbf{x}-\mathbf{y}\right),$$

$$\langle F_i(\mathbf{x})F_j^*(\mathbf{y})\rangle = S_{ij}\left(\frac{\mathbf{x}+\mathbf{y}}{2},\mathbf{x}-\mathbf{y}\right),i,j=1,2,3. \tag{13}$$

The acoustic pressure is given by

$$p(\mathbf{x},\omega) = i\omega^{-1}\int [B(\mathbf{y})\tilde{G}(\mathbf{y},\mathbf{x},\omega) - \mathbf{F}(\mathbf{y})\cdot\tilde{\mathbf{g}}(\mathbf{y},\mathbf{x},\omega)]d^3\mathbf{y}, \tag{14}$$

where the GF $G(\mathbf{x},\mathbf{y},\omega)$ and the Green's vector $\mathbf{g}(\mathbf{x},\mathbf{y},\omega)$ have the meaning of the frequency spectra of the acoustic pressure and oscillatory particle displacement at point $\mathbf{x}$ due to a point source of volume velocity with $\mathbf{F}=0$ and $B(\mathbf{x},t)=\delta(\mathbf{x}-\mathbf{y})\delta'(t)$ [13]. At high frequencies, $\mathbf{g}(\mathbf{y},\mathbf{x},\omega)=\Sigma_n \mathbf{a}_n(\mathbf{y},\mathbf{x})\exp[i\omega\varphi_n(\mathbf{y},\mathbf{x})]$, where $\mathbf{a}_n$ are slowly varying complex amplitudes, while φ_n are the same eikonals as in Eq. (4) for the GF [16].

From Eqs. (13) and (14), for the CCF of the acoustic pressure field we find

$$\begin{aligned}\langle p(\mathbf{x}_1,\omega)p^*(\mathbf{x}_2,\omega)\rangle &= \omega^{-2}\int\int\Big\{\tilde{G}\Big(\mathbf{y}_1+\frac{\mathbf{y}_2}{2},\mathbf{x}_1\Big)\tilde{G}^*\Big(\mathbf{y}_1-\frac{\mathbf{y}_2}{2},\mathbf{x}_2\Big)Q(\mathbf{y}_1,\mathbf{y}_2)\\ &\quad-\tilde{G}\Big(\mathbf{y}_1+\frac{\mathbf{y}_2}{2},\mathbf{x}_1\Big)\tilde{g}_j^*\Big(\mathbf{y}_1-\frac{\mathbf{y}_2}{2},\mathbf{x}_2\Big)R_j(\mathbf{y}_1,\mathbf{y}_2)\\ &\quad-\tilde{G}^*\Big(\mathbf{y}_1-\frac{\mathbf{y}_2}{2},\mathbf{x}_2\Big)\tilde{g}_j\Big(\mathbf{y}_1+\frac{\mathbf{y}_2}{2},\mathbf{x}_1\Big)R_j(\mathbf{y}_1,-\mathbf{y}_2)\\ &\quad+\tilde{g}_i\Big(\mathbf{y}_1+\frac{\mathbf{y}_2}{2},\mathbf{x}_1\Big)\tilde{g}_j^*\Big(\mathbf{y}_1-\frac{\mathbf{y}_2}{2},\mathbf{x}_2\Big)S_{ij}(\mathbf{y}_1,\mathbf{y}_2)\Big\}d^3\mathbf{y}_1d^3\mathbf{y}_2,\end{aligned} \tag{15}$$

where summation over repeated indices i and j is implied. By approximating $\mathbf{g}(\mathbf{y}_1 \pm \mathbf{y}_2/2,\mathbf{x})$ in Eq. (15) similarly to the GF approximation (5) and following the same reasoning as in the derivation of Eq. (8), we find

$$\begin{aligned}\langle p(\mathbf{x}_1,\omega)p^*(\mathbf{x}_2,\omega)\rangle &\approx \omega^{-2}\int\{\tilde{G}(\mathbf{y},\mathbf{x}_1)\tilde{G}^*(\mathbf{y},\mathbf{x}_2)Q_1(\mathbf{y},\mathbf{0}) - \tilde{G}(\mathbf{y},\mathbf{x}_1)\tilde{g}_j^*(\mathbf{y},\mathbf{x}_2)\\ &\quad\times R_j^{(1)}(\mathbf{y},\mathbf{0}) - \tilde{G}^*(\mathbf{y},\mathbf{x}_2)\tilde{g}_j(\mathbf{y},\mathbf{x}_1)R_j^{(1)*}(\mathbf{y},\mathbf{0})\\ &\quad+\tilde{g}_i(\mathbf{y},\mathbf{x}_1)\tilde{g}_j^*(\mathbf{y},\mathbf{x}_2)S_{ij}^{(1)}(\mathbf{y},\mathbf{0})\}d^3\mathbf{y},\end{aligned} \tag{16}$$

when either $\mathbf{x}_1=\mathbf{x}_2$ or $k|\mathbf{x}_1-\mathbf{x}_2| \gg 1$. Here $Q_1(\mathbf{y},\mathbf{q})$, $\mathbf{R}^{(1)}(\mathbf{y},\mathbf{q})$, and $S_{ij}^{(1)}(\mathbf{y},\mathbf{q})$ are spatial spectra of $Q(\mathbf{y},\mathbf{y}_1)$, $\mathbf{R}(\mathbf{y},\mathbf{y}_1)$, and $S_{ij}(\mathbf{y},\mathbf{y}_1)$, see Eq. (7). Equation (16) coincides with the CCF of the field due to delta-correlated sources which follows immediately from Eq. (15) when $Q(\mathbf{y},\mathbf{y}_1)=Q_1(\mathbf{y},\mathbf{0})\delta(\mathbf{y}_1)$, $\mathbf{R}(\mathbf{y},\mathbf{y}_1)=\mathbf{R}^{(1)}(\mathbf{y},\mathbf{0})\delta(\mathbf{y}_1)$, and $S_{ij}(\mathbf{y},\mathbf{y}_1)=S_{ij}^{(1)}(\mathbf{y},\mathbf{0})\delta(\mathbf{y}_1)$.

Using representations [13] of wave fields due to sources within the fluid and the solid in terms of Green's functions, vectors, and tensors and approximating the Green's vectors and tensors as in Eq. (5), it is straightforward to verify the validity of the above results for waves in a coupled solid-fluid flow system. Moreover, the derivations presented above apply also to electromagnetic waves in motionless dielectrics as long as wave absorption per wavelength remains small.

V. DISCUSSION

It is, of course, expected that the differences between statistical properties of wave fields due to delta-correlated sources and sources with a finite correlation length l_2 vanish when l_2 is small compared to all other relevant spatial scales of the problem, including the wavelength. Here we have found that long-range cross correlation of high-frequency fields due to correlated sources approximates the CCF of the field due to certain "effective" delta-correlated sources under much weaker conditions, namely, when l_2 is small compared to the linear dimensions of the Fresnel zone. When a point $\mathbf{y}$ moves away from the receivers along a ray connecting $\mathbf{x}_1$ and $\mathbf{x}_2$, the dimensions of the Fresnel zone in the directions transverse to the ray increase. In a typical situation, where $|\mathbf{y}-\mathbf{x}_{1,2}|$ is either large or of the order of $|\mathbf{x}_1-\mathbf{x}_2|$ for the noise sources which are responsible for the main contribution to the CCF, it is sufficient to require that l_2 is small compared to the linear dimensions (in the directions transverse to the ray path) of the Fresnel zone for propagation between points $\mathbf{x}_1$ and $\mathbf{x}_2$.

It is important to emphasize that the wave fields due to correlated and uncorrelated sources generally remain rather different when their CCFs become indistinguishable. Compare, for example, noise fields in a homogeneous fluid at rest generated by either delta-correlated or finite random sources distributed on the same surface $\partial\Omega$. Let $\partial\Omega$ be a plane $\mathbf{x}\cdot\mathbf{N}=0$, $\mathbf{N}=(0,0,1)$ and choose the parameters α and β as Cartesian coordinates in the plane $\partial\Omega$. The main energy characteristic of the noise field at a point $\mathbf{y}$, usually referred to as the ray intensity, is the average power $I(\mathbf{y},\mathbf{s})$ radiated into a unit solid angle in the direction of the unit vector $\mathbf{s}=(s_1,s_2,s_3)$. Using the method of stationary phase, it is easy to show that the ratio χ of the ray intensities due to finite and delta-correlated sources is $\chi(\mathbf{y},\mathbf{s})=E_1(\mathbf{x}_0;ks_1,ks_2)/d(\mathbf{x}_0)$, where d is the density of the delta-correlated sources and $\mathbf{x}_0=\mathbf{y}-\mathbf{s}(\mathbf{N}\cdot\mathbf{y})/s_3$. For instance, when $E(\mathbf{y};\mathbf{y}_1)=\pi^{-1}l_2^{-2}d(\mathbf{y})\exp\{-[\mathbf{y}_1-(\mathbf{y}_1\cdot\mathbf{N})\mathbf{N}]^2/l_2^2\}$, we have $E_1(\mathbf{y};0,0)=d(\mathbf{y})$ and $\chi=\exp[-k^2l_2^2(1-s_3^2)/4]$. While the two noise fields have the same long-range CCFs, the ray intensity away from the normal to $\partial\Omega$ is much smaller in the case of finite sources unless their length scale l_2 is small compared to the wavelength $2\pi/k$.

The physical reason behind the weak sensitivity of the CCF to the correlation range of noise sources is very simple. The phase difference of the contributions to the CCF from a source at the stationary point $\mathbf{x}_0$ and a point $\mathbf{x}$ in the vicinity of $\mathbf{x}_0$ is of the second order in $\mathbf{x}-\mathbf{x}_0$. All the noise sources within the Fresnel zone on $\partial\Omega$ contribute coherently to the CCF whether these are correlated or not. On the other hand, the phase difference of the contributions of sources at points $\mathbf{x}$ and $\mathbf{x}_0$ into the ray intensity in the direction $\mathbf{s}$ approximately equals $[\mathbf{s}-(\mathbf{N}\cdot\mathbf{s})\mathbf{s}]\cdot(\mathbf{x}-\mathbf{x}_0)$ and is linear in $\mathbf{x}-\mathbf{x}_0$, except for the ray intensity in the direction normal to $\partial\Omega$. Contributions due to coherent sources interfere destructively in the off-normal directions. Therefore, the noise directivity is affected by the source correlation ranges l_2 of the order of the wavelength and greater.

When planning an experiment on passive imaging, one can avoid the influence of noise sources correlation on the GF emergence from the noise CCF by choosing sufficiently large separation between receivers. Let the noise sources with correlation range l_2 be distributed on a surface $\partial\Omega$, the minimum distance between $\partial\Omega$ and receivers at points $\mathbf{x}_1$ and

$\mathbf{x}_2$ be of the order of $|\mathbf{x}_2-\mathbf{x}_1|$ or larger, and $k|\mathbf{x}_2-\mathbf{x}_1|\gg 1$. Here k is a representative value of wave number in an inhomogeneous medium. As shown in Sec. III, the effect on the CCF of the sources correlation range is negligible when $|\mathbf{x}_2-\mathbf{x}_1| \gg kl_2^2$. Increasing the receiver separation $|\mathbf{x}_2-\mathbf{x}_1|$ also helps to improve accuracy of retrieval of the eikonals $\varphi(\mathbf{x}_2,\mathbf{x}_1)$ and $\varphi(\mathbf{x}_1,\mathbf{x}_2)$ from the measured CCF [17,18] as well as accuracy of the eikonal inversion for path-averaged sound speed and flow velocity. However, larger $|\mathbf{x}_2-\mathbf{x}_1|$ require longer noise accumulation times necessary for the GF emergence from noise [7].

Many sources of acoustic noise (ships at sea, vehicles on the road, aircraft, etc.) continuously change their position in time. In the resulting noise field, motion of the noise sources may manifest itself as their spatial coherence. If a delta-correlated source radiates a broadband signal with a correlation time τ and moves with a deterministic or random speed of the order of V, signals emitted at points $\mathbf{y}_1$ and $\mathbf{y}_2$ along the source trajectory at distances $|\mathbf{y}_1-\mathbf{y}_2|\sim V\tau$ will be correlated, just like in the case of an extended stationary source with correlation length $l_2=V\tau$.

To give a quantitative example of a relation between statistical descriptions of moving and stationary sources, consider sound sources with density $B(\mathbf{x},t) = m(t)n[\mathbf{x}-\mathbf{r}_0(t)]$, where m and n are statistically homogeneous random functions: $\langle m(t)\rangle=0$, $\langle n(\mathbf{x})\rangle =0$, $\langle m(t_1)m(t_2)\rangle=M(t_1-t_2)$, $\langle n(\mathbf{x}_1)n(\mathbf{x}_2)\rangle=N(\mathbf{x}_1-\mathbf{x}_2)$, with representative scales of the correlation functions M and N dependence on their arguments being τ and l, respectively. Let relative variations of the source velocity $\mathbf{v}_0(t)=d\mathbf{r}_0/dt$ be small over times $O(\tau)$. Then

$$\langle B(\mathbf{x}_1,t_1)B(\mathbf{x}_2,t_2)\rangle = M(t_1-t_2)N[\mathbf{x}_1-\mathbf{x}_2-(t_1-t_2)\mathbf{v}_0], \tag{17}$$

where $\mathbf{v}_0$ can be evaluated at $t=(t_1+t_2)/2$. Note that the source motion extends the support of the spatial correlation functions by about $\pm v_0\tau$ along the direction of the motion. Assuming that the correlation function N is isotropic and Gaussian and M is Gaussian, it follows from Eq. (17) that the source motion results in an increase of the frequency band by the factor $(1+v_0^2\tau^2/l^2)^{1/2}$ and an increase of the correlation length l_2 of the function Q in Eq. (2) from l to $(l^2+v_0^2\tau^2)^{1/2}$ in the direction of the source motion. According to results of Secs. II and III, motion of the noise sources will not affect emergence of deterministic GFs from the noise CCF in inhomogeneous and moving media as long as the effective source correlation length $(l^2+v_0^2\tau^2)^{1/2}$ remains small compared to the Fresnel zone size. With stationary receivers and τ being sufficiently small, the CCF approximates the GFs that correspond to motionless sources, even when the actual noise sources move at supersonic speeds. Similar conclusions were first reached, from other considerations, by Sabra [19] who studied spatially delta-correlated noise sources moving in a homogeneous fluid.

VI. CONCLUSION

We have demonstrated that all theoretical results previously established for the two-point correlation function of high-frequency wave fields due to delta-correlated sources remain valid for long-range cross-correlation of wave fields due to extended random sources as long as the correlation length of the latter is small compared to the size of the Fresnel zone at wave propagation between the two points. Consequently, the CCF of ambient noise generated by extended random sound sources in moving fluids or fluid-solid systems, as in the case of delta-correlated sources [5,13], approximates the sum of the GFs, which describe wave propagation in opposite directions between the two points. Measurement of the CCF allows one to determine acoustic travel times in opposite directions, which are normally obtained in reciprocal transmissions experiments employing acoustic transceivers located at the two points and are used as input data for a tomographic retrieval of the sound speed and current velocity fields [20,21]. In application to motionless media, our results provide a theoretical justification for using CCFs of nondiffuse noise fields due to extended sources [6–11] in the imaging of Earth's crust.

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